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POINT SOURCE RADIATION PATTERN SYNTHESIS BY ITERATIVE TECHNIQUE--ETC(U)

FEB 77 J PERINI, J CHOU

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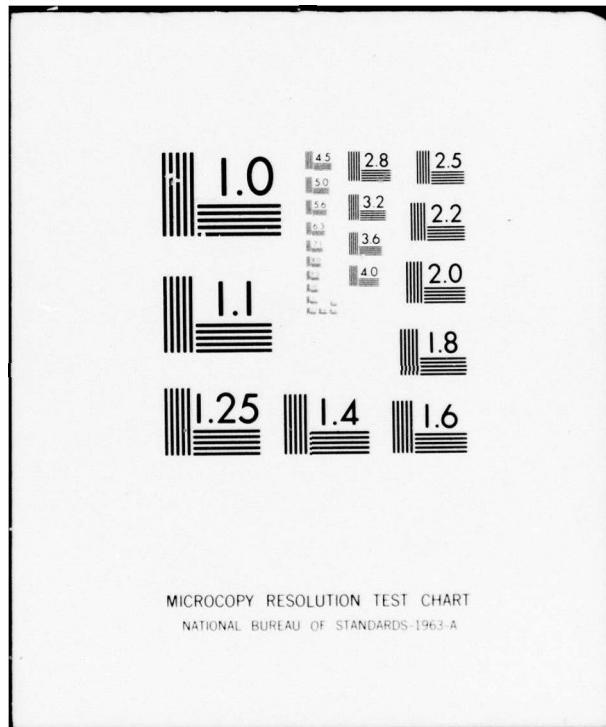
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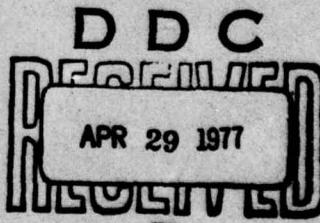
RADC-TR-77-33  
Phase Report  
February 1977



POINT SOURCE RADIATION PATTERN SYNTHESIS  
BY ITERATIVE TECHNIQUES

Syracuse University

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## PREFACE

This effort was conducted by Syracuse University under the sponsorship of the Rome Air Development Center Post-Doctoral Program for NAVSEC. Mr. Tony Testa of NAVSEC was the task project engineer and provided overall technical direction and guidance. The authors of this report are Dr. Jose Perini and Jason Chou.

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POINT SOURCE RADIATION PATTERN SYNTHESIS  
BY ITERATIVE TECHNIQUES

1. Introduction

Many different synthesis techniques for antenna radiation patterns have been developed over the years [1-6]. Unfortunately, most of these techniques apply only to a restricted class of problems. As one example of the case in point, usually these techniques either apply to uniformly or to nonuniformly spaced arrays. The reason for this distinction is rooted in the fundamental different functional dependence of the array radiation pattern on the elements' amplitudes and spacings. The method proposed in this paper is independent of the functional dependence of the array parameters and therefore treats them on an equal basis. This makes the method very general and applicable to a truly large class of problems. Furthermore, the method allows the introduction of a variety of linear and nonlinear constraints in the synthesis, giving the designer the opportunity to simplify the antenna construction. As an example, the element currents can be constrained to have the same magnitude which simplifies the feeding system considerably. The method is based on the well-known steepest descent technique [7], implemented in the form of a computer program. Many other optimization methods, which are available in the literature, could also have been applied to this problem with various degrees of success and complexity [8-12]. The steepest descent has been chosen due to its simplicity in implementing constraints and minimal programming effort.

## 2. The Steepest Descent Technique

In order to explain this technique let us assume that we have a function of two variables  $z = f(x,y)$ . This is the equation of a surface in the three-dimensional space  $(x,y,z)$ . Our problem is to find the position  $(x_m, y_m)$  of the minimum  $z_m$  on this surface. The situation is illustrated in Fig. 1. The way we can proceed is to choose any initial values for  $(x,y)$ , say  $(x^0, y^0)$ , find the corresponding point  $z^0$  on the surface and, starting from  $z^0$ , "walk" always in the direction of the steepest descent until the bottom of the valley is found. In mathematical language the direction of the steepest descent is that of the gradient of  $f(x,y)$  or  $\nabla z$ . We can then go from  $(x^0, y^0)$  to the next point  $(x^1, y^1)$  by computing

$$x^1 = x^0 - k \nabla_x z^0 \quad (1)$$

$$y^1 = y^0 - k \nabla_y z^0 \quad (2)$$

where  $k$  is the gain constant which controls the convergence rate of the method,  $\nabla_x z^0$  is the  $x$  component of  $\nabla z$  computed at  $(x^0, y^0)$ , etc. Once  $(x^1, y^1)$  is found we compute  $z^1 = f(x^1, y^1)$  and compare it with  $z^0$  to see if we actually have proceeded down hill. We then continue in a similar fashion until the bottom of the valley  $z_m$  is found. The details of this method and more sophisticated ones are found in the literature [7-12].

Let us now apply this technique to our problem. In order to obtain results which are general enough let us write our equations for the planar array case where each element may have a different radiation pattern  $P_i(\theta)$ , oriented arbitrarily as shown in Fig. 2. The pair  $(\phi_i, d_i)$

determines the position of the  $i^{\text{th}}$  element, the pair  $(I_i, \alpha_i)$  specifies the magnitude and phase of the element excitation while  $\gamma_i$  determines the orientation of the element radiation pattern. Let us see how we would particularize this general representation for a few of the different cases we have in mind. For a linear array we could position all elements on the  $x$  axis. Therefore  $\phi_i = 0^\circ$  or  $180^\circ$  depending on whether the element lies to the right or to the left of the origin. If all elements have omnidirectional patterns  $\gamma_i$  may have any arbitrary value (zero for example). If the patterns are all pointing broadside (in the  $y$  direction) then  $\gamma_i = 90^\circ$ . In the case of an AM broadcasting array, as all towers have an omnidirectional pattern  $P_i(\theta) = \text{constant} = 1$ ,  $\gamma_i = 0$  and  $(\phi_i, d_i)$  will determine the position of each element.

The radiation pattern of the planar array consisting of  $N$  excited elements of Fig. 2 can be written as

$$P(\phi) = \sum_{i=1}^N P_i(\phi - \gamma_i) I_i e^{-j[\frac{2\pi d_i}{\lambda} \cos(\phi - \phi_i) - \alpha_i]} \quad (3)$$

Let  $P_s(\phi)$  be the selected pattern of interest. Usually only the magnitude  $|P_s(\phi)|$  of the pattern is specified. The error between the computed pattern  $|P(\phi)|$  and  $|P_s(\phi)|$  for any choice of the array parameters is given by

$$D(\phi) = |P(\phi)| - |P_s(\phi)| \quad (4)$$

if we choose  $M$  directions  $\phi_j$ ,  $j=1, 2, 3, \dots, M$ , at which we want to minimize the error  $D(\phi)$ , we can define the error over the entire synthesis range as

$$E = \sum_{j=1}^M W(\phi_j) D^2(\phi_j) = \sum_{j=1}^M W(\phi_j) [|P(\phi_j)| - |P_s(\phi_j)|]^2 \quad (5)$$

The reason for the square in the above equation is to make sure that when  $E$  is a minimum each component of the sum is also a minimum and therefore we have the best approximation of  $|P_s(\phi)|$  by  $|P(\phi)|$  in the mean.  $W(\phi_j)$  is a weighting function that allows the synthesis precision to be changed over certain ranges of  $\phi$ . Its use will be explained in Section 3 in connection with specific examples. Note now that  $E$  is a function of all the array parameters

$$E = E(\underline{\phi}_1, \underline{d}_1, \underline{I}_1, \underline{\alpha}_1, \underline{\gamma}_1) \quad (6)$$

where  $\underline{\phi}_1$  is used to represent the set of variables  $\phi_1, \phi_2, \phi_3, \dots, \phi_N$  and etc. Equation (6) is of the type  $z = f(x, y)$ , the only difference being in the number of variables. Instead of  $(x, y)$  we have now  $5N$  variables for  $N$  elements each with 5 variables.  $E$  is therefore a "surface" in a  $5N$  dimensional space. The steepest descent technique can then be used to find the minimum  $E_m$  on this surface. We proceed in the usual way. We start with an initial guess for the parameters  $(\underline{\phi}_1^0, \underline{d}_1^0, \underline{I}_1^0, \underline{\alpha}_1^0, \underline{\gamma}_1^0)$  compute the gradient at this point  $\nabla E^0$  and then compute the new values for the parameters  $(\underline{\phi}_1^1, \underline{d}_1^1, \underline{I}_1^1, \underline{\alpha}_1^1, \underline{\gamma}_1^1)$  using equations (1) and (2) as

$$\begin{aligned} \underline{\phi}_1^1 &= \underline{\phi}_1^0 - k_\phi \nabla_\phi E^0 \\ \underline{d}_1^1 &= \underline{d}_1^0 - k_d \nabla_d E^0 \\ \underline{I}_1^1 &= \underline{I}_1^0 - k_I \nabla_I E^0 \\ \underline{\alpha}_1^1 &= \underline{\alpha}_1^0 - k_\alpha \nabla_\alpha E^0 \\ \underline{\gamma}_1^1 &= \underline{\gamma}_1^0 - k_\gamma \nabla_\gamma E^0 \end{aligned} \quad (7)$$

In these equations  $\nabla E^0$  are the components of  $\nabla E$  corresponding to the variables  $\underline{\phi}_i$ , evaluated at the point  $(\underline{\phi}_i^0, \underline{d}_i^0, \underline{I}_i^0, \underline{\alpha}_i^0, \underline{y}_i^0)$  and  $k_{\phi}$  is called the gain constant for the same variables. A similar interpretation applies to all other equations. A word of caution should be said about the gain constants. First of all we only follow the steepest descent if all  $k$ 's of (7) are equal. Second, if the  $k$ 's are chosen to be too small the method will converge very slowly and if the  $k$ 's are too large the process may even diverge. These two possibilities are illustrated in Figs. 3 and 4 respectively. As seen in Fig. 3 the progress made in each step is very small while in Fig. 4 the large value of  $k$  causes the successive steps to oscillate around the minimum. The choice of  $k$  is very critical and has been treated extensively in the literature [7,9,10,11,12]. The reason for the use of different  $k$ 's in (7) is that it allows the implementation of certain constraints very easily. For example if we desire to maintain the currents all constant and equal to the initial guess we just have to set  $k_I = 0$ .

Any types of constraints are easily implemented in this method. Figure 5 shows the case in which the boundary  $CC'$  cannot be crossed. At every step of the process we check to see if the values of  $x$  and  $y$  are such that they represent a point in the Forbidden Region such as  $P^1$ . If this is the case the value of  $k$  is adjusted such that we stop just on the boundary at  $P_c^1$ . If at  $P_c^1$  the gradient still points to the Forbidden Region we are forced to "walk" on the boundary as in  $P^2$  otherwise we move to the Permissible Region and proceed in the usual way.

### 3. Application to some Specific Synthesis Problems

Let us now apply the above ideas to some specific cases.

Example 1 - Assume we want to synthesize the pattern specified by the eighteen small circles ( $M = 18$ ) in Fig. 6 for an AM Broadcasting Antenna. Therefore the elements are omnidirectional implying  $P_i(\theta) = 1$  and  $\gamma_i = 0$ . We want to use 3 elements ( $N = 3$ ) and do not have any specific requirement in the elements position. The synthesis resulted in an almost linear array in the  $270^\circ$  radial. The element spacing is approximately  $\lambda/4$  and the currents are almost equal. We could have performed the synthesis imposing the constraint that the currents be all equal and that the array is linear. Instead of doing this let us proceed to a more interesting example.

Example 2 - The desired radiation pattern is specified by the twenty-two small circles ( $M = 22$ ) of Fig. 7. Let us try to synthesize this pattern with four elements ( $N=4$ ), each having the radiation pattern shown in Fig. 8 ( $\cos^2 \theta$  with 10% back lobe), and let us assume that the elements are fixed in position and orientation as shown in the insert of Fig. 7. Therefore the variables in the problem are only the current amplitudes since we desire to maintain the phase constant too. With so many constraints we should not expect to obtain a good fit as shown in Fig. 7.

Example 3 - Let us now use the same data of Example 2 but let all parameters change freely. The result is shown in Fig. 9 with a consequent marked improvement in the fit. The array parameters as well as the elements are also shown in Fig. 9.

Example 4 - One of the authors has developed the theory of skewed arrays [13,14]. Let us try to obtain a pattern as smooth as possible,  $|P_s(\theta)| = 1$ , using four elements with the radiation pattern shown in Fig. 8 but with no back lobe. The elements are constrained to be in the radials  $0^\circ, 90^\circ, 180^\circ, 270^\circ$ , the distance to the origin being the only parameter that changes. Each element has its pattern oriented in a skew fashion as shown in the insert of Fig. 10. The amplitudes are to be equal to one and the phases equal to zero. The result is shown by the solid line in Fig. 10 and represents an improvement over the theoretical result (shown dashed) obtained previously [14]. The inter element distance is  $3.64 \times \sqrt{2} \approx 5.2\lambda$  instead of  $5\lambda$  as theoretically predicted.

Example 5 - The same problem of Example 4 is used here with the difference that now each element has a pattern like Fig. 8, including the 10% back lobe. The result is shown in Fig. 11 which is still better than the theoretical one [14]. The inter element separation is the same as that of Example 4.

Example 6 - In this example we will try to synthesize the vertical pattern of a TV transmitting antenna. The specified pattern is shown in Fig. 12 by the small circles. Each element has the radiation pattern shown dashed. The antenna feed system is shown in the insert. The following constraints are imposed:

- 1 - All elements should have equal current

2 - The element spacing should be greater than  $7.5\lambda$ , since each element has approximately  $7\lambda$  aperture. The two central elements should be  $8\lambda$  apart to facilitate the feed system construction.

3 - As the elements are capacitively coupled to the line it is desired that the computed phases of each element be given by its physical position along the coaxial line. This is therefore a constraint relating the element position to its phase.

We would like to take this opportunity to illustrate here how the weighting function W can be chosen. For this particular case W was selected in the following way:

1 - In the main beam portion  $W = 5$  since we want the main beam to be synthesized with good accuracy. This is equivalent to specifying each point in the main beam five times.

2 - In the "UP" region W was chosen equal to one or zero depending on whether the computed pattern exceeds or not the specified one. This implies that we do not care about the particular shape of the pattern in this region as long as it is below the specified level.

3 - In the "DOWN" region we do not want the pattern to be below the specified one since this will result in deficient station coverage. On the other hand if the synthesized pattern is too much above the specified one the gain will be decreased, therefore we choose W to be equal to two or one half depending on

whether the computed pattern is below or above the specified one. This means that a deviation below the specification weights four times more than a deviation above. The result is shown in Fig. 12.

Example 7 - If in Example 6 we relax the condition that the currents have to be of the same magnitude, then a better fit is obtained as shown in Fig. 13. In this case the power split between the upper and the lower parts of the antenna is 47:53. It would be a simple matter to resynthesize this pattern with the constraint of a 50:50 power split. All that has to be done is to compute one of the currents from the other five under a 50:50 power split condition.

#### 4. Conclusions and Recommendations

It has been shown here that the steepest descent technique, when implemented through a computer program, is a very powerful tool for the synthesis of radiation patterns. The method is extremely flexible and can handle almost any kind of linear and nonlinear constraints. It provides results that cannot be obtained by any other synthesis method known to the authors.

The mutual coupling among the antennas has been neglected. This effect can be taken into consideration by using an equivalent point source representation of each antenna in its actual environment. This will be the subject of a subsequent report.

In order to use this computer program, which is described in Appendix A, the user must have some knowledge of the system for which

it was written (RADC H-645) and be versed in computer programming. The reason for this is that the main purpose in the minds of the authors was to test the technique. The next step, then, after mutual couplings are taken into account, is to make the program much more user oriented.

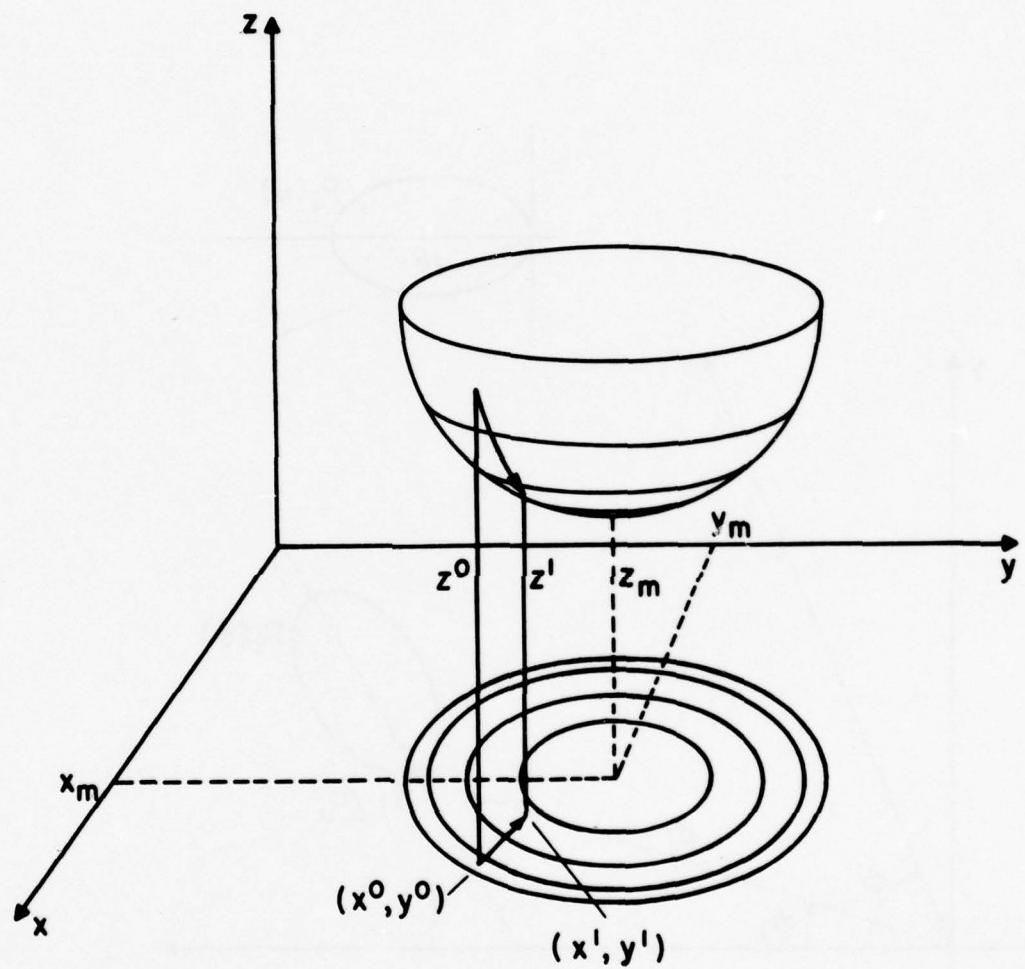


Fig. 1 - The Steepest Descent Method

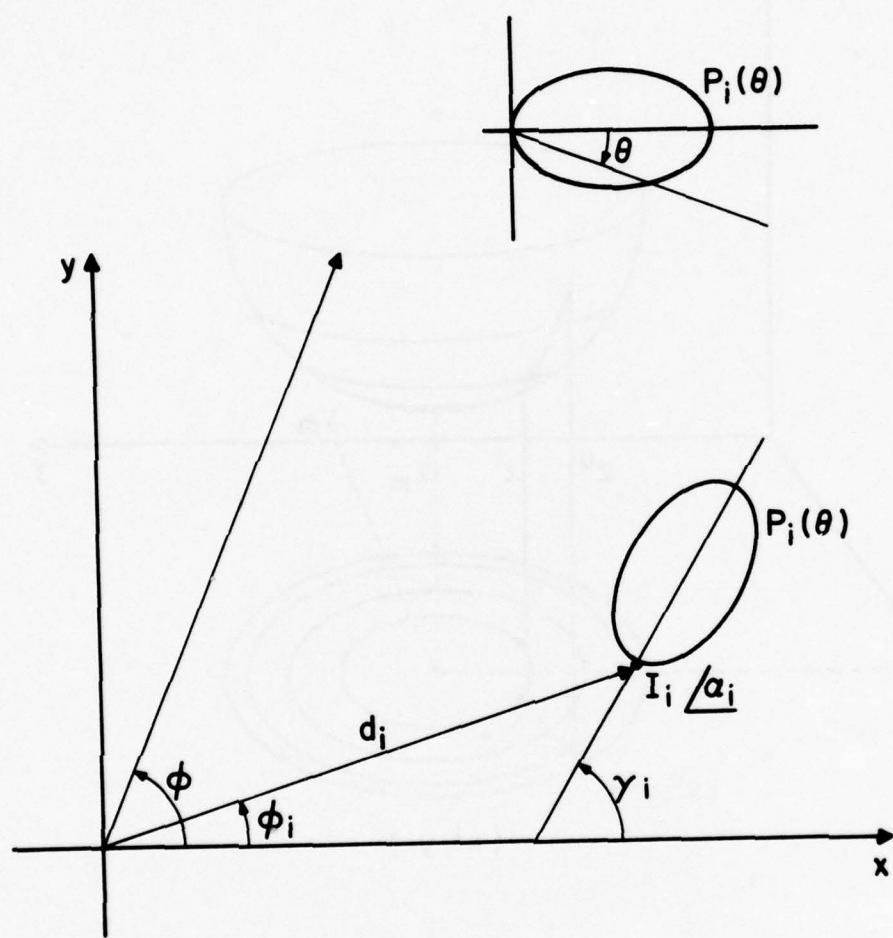


Fig. 2 - Planar Array Variables

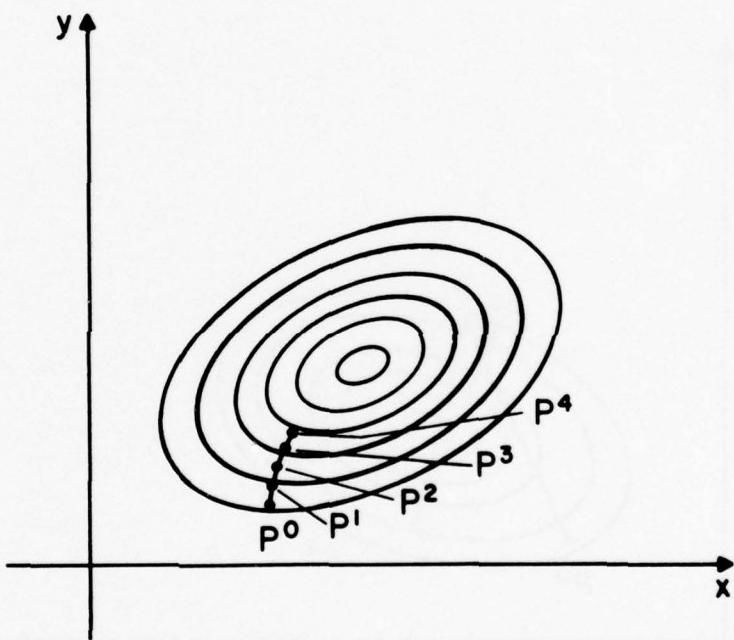


Fig. 3 - Slow Progress for Small Gain Constant

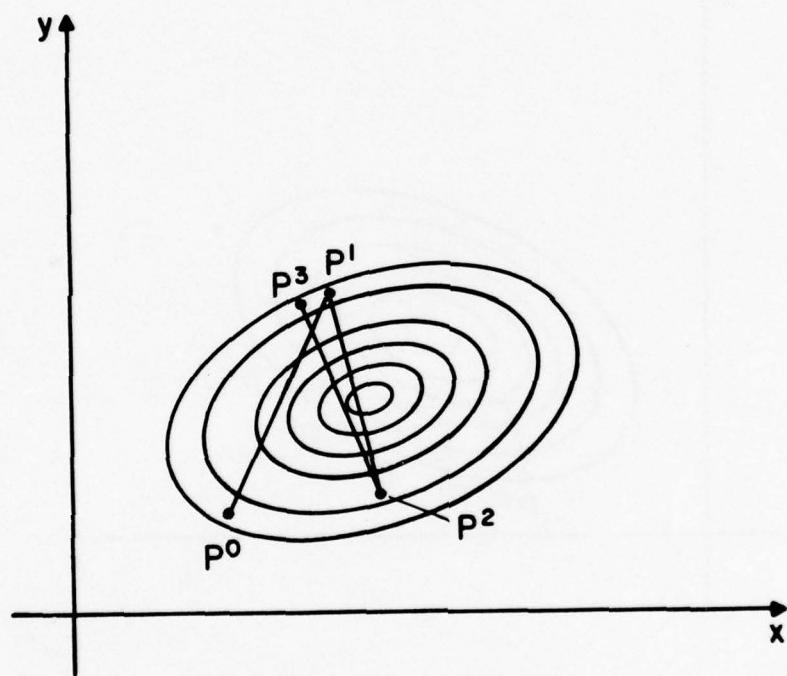


Fig. 4 - Oscillation for Large Gain Constant

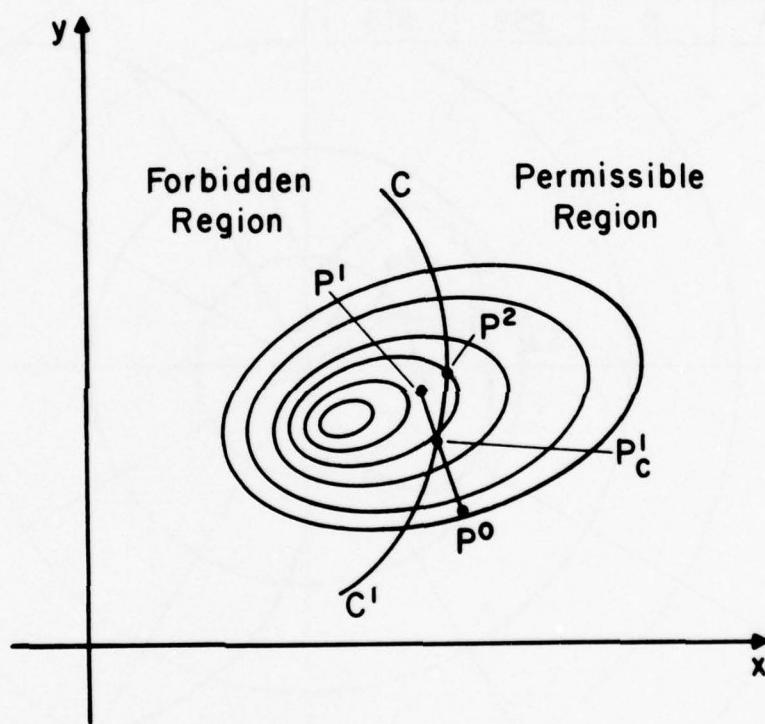


Fig. 5 - Implementation of Constraint Boundary  $CC'$

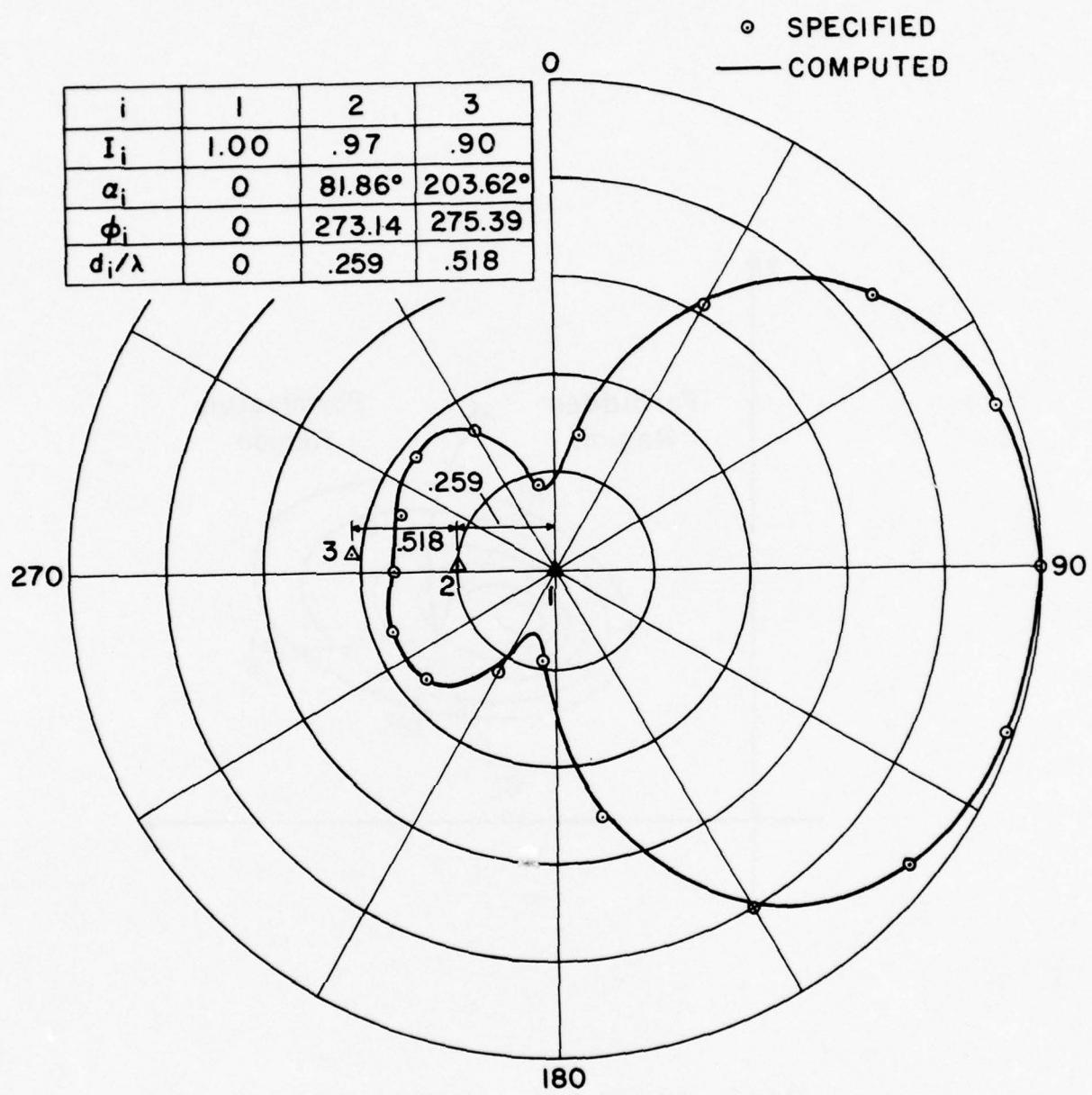


Fig. 6 - AM Broadcasting Antenna Pattern

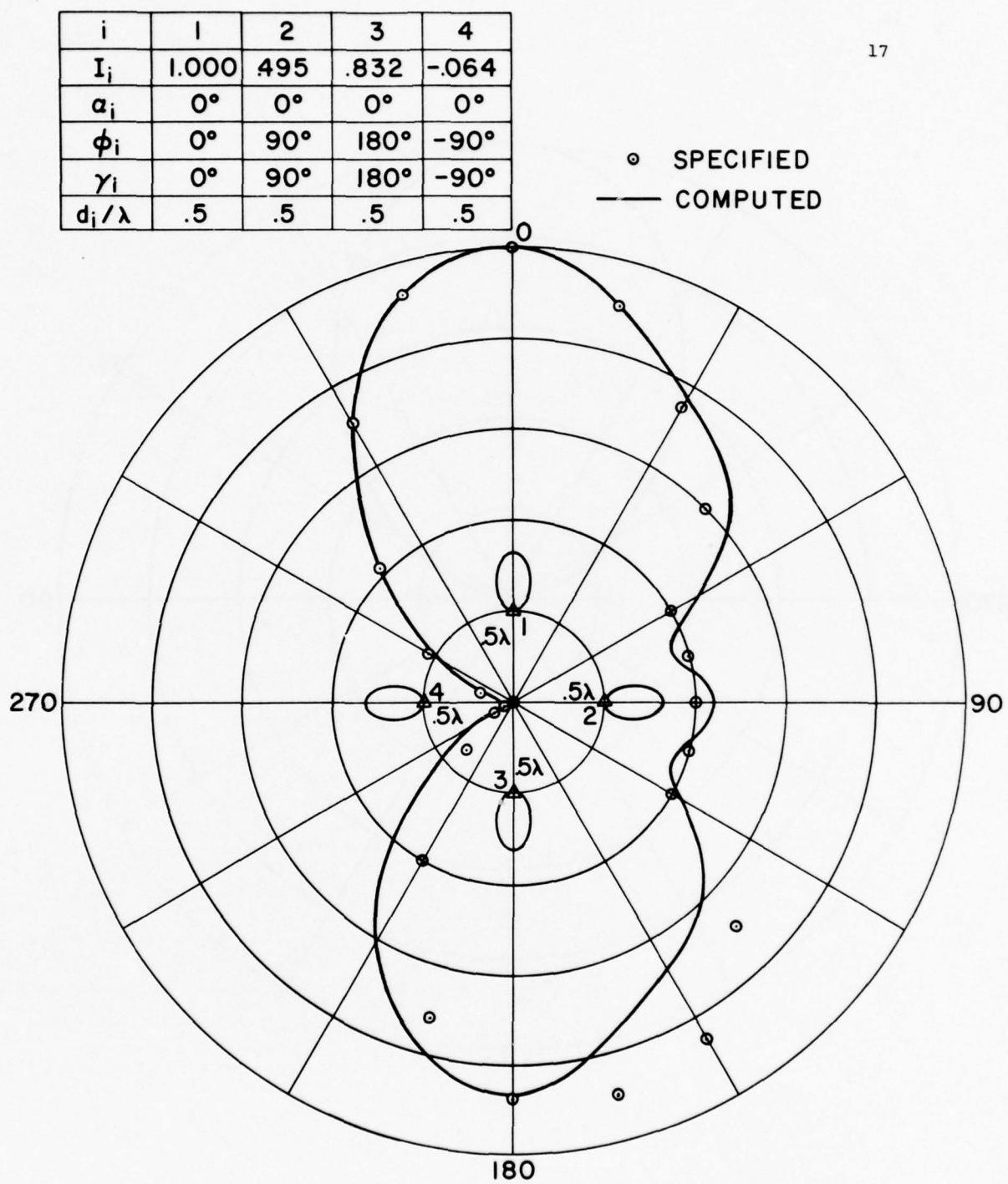


Fig. 7 - Synthesis With Variable Amplitude Only

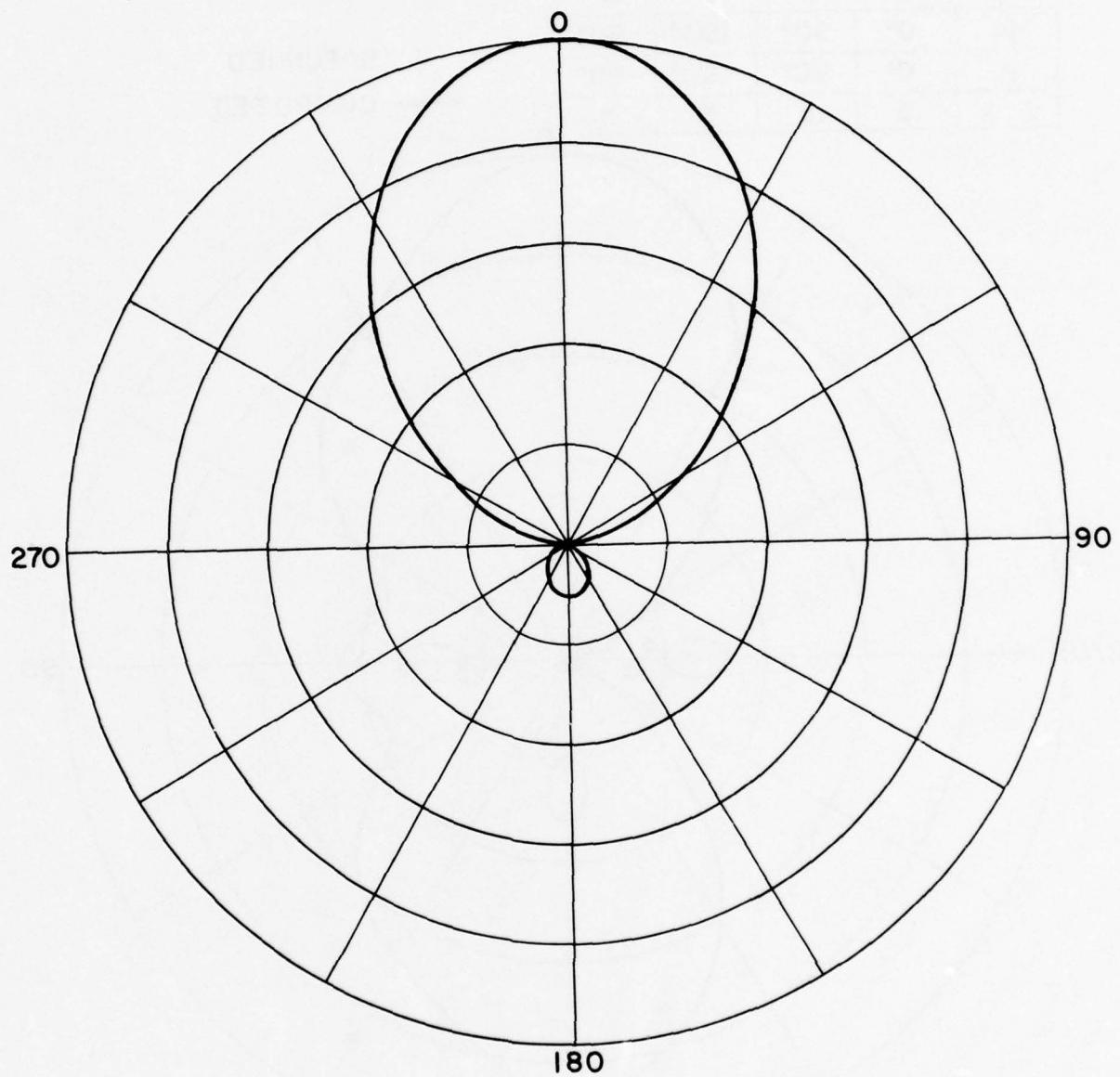


Fig. 8 -  $\cos^2$  Pattern With 10% Back Lobe

$i$	1	2	3	4
$I_i$	1.000	.519	.854	-.054
$\alpha_i$	$0^\circ$	$19.3^\circ$	$-2.0^\circ$	$10.2^\circ$
$\phi_i$	$-8^\circ$	$91.2^\circ$	$179.2^\circ$	$-89.9^\circ$
$\gamma_i$	$-2.0^\circ$	$92.6^\circ$	$169.3^\circ$	$-90.4^\circ$
$d_i/\lambda$	.497	.511	.414	.506

○ SPECIFIED  
— COMPUTED

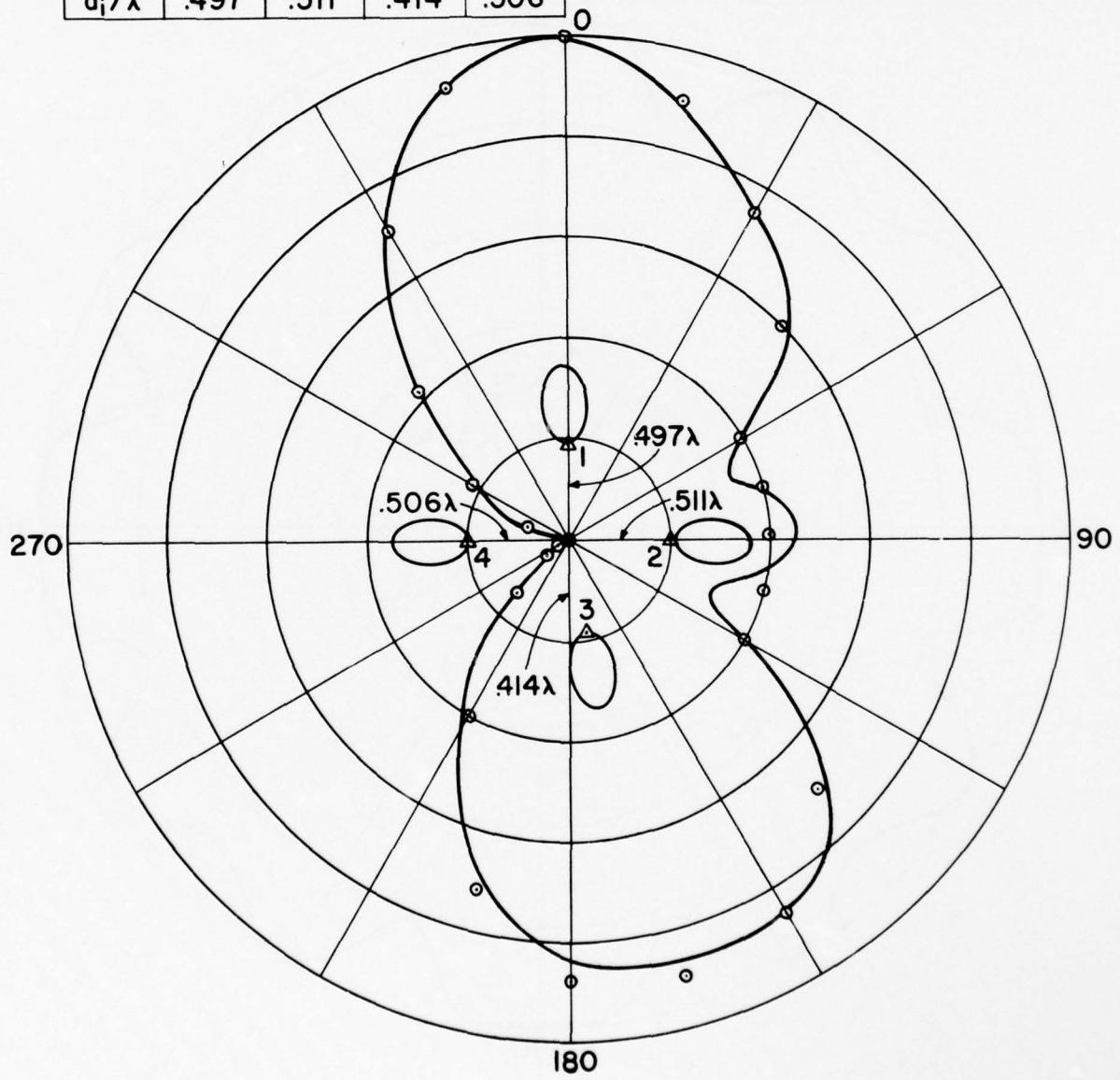


Fig. 9 - Synthesis With all Parameters Free to Vary

i	1	2	3	4
$I_i$	1.060	1.060	1.060	1.060
$\alpha_i$	0°	0°	0°	0°
$\phi_i$	0°	90°	180°	-90°
$\gamma_i$	90°	180°	-90°	0°
$d_i/\lambda$	3.64	3.64	3.64	3.64

----- THEORETICAL  
— OPTIMIZED

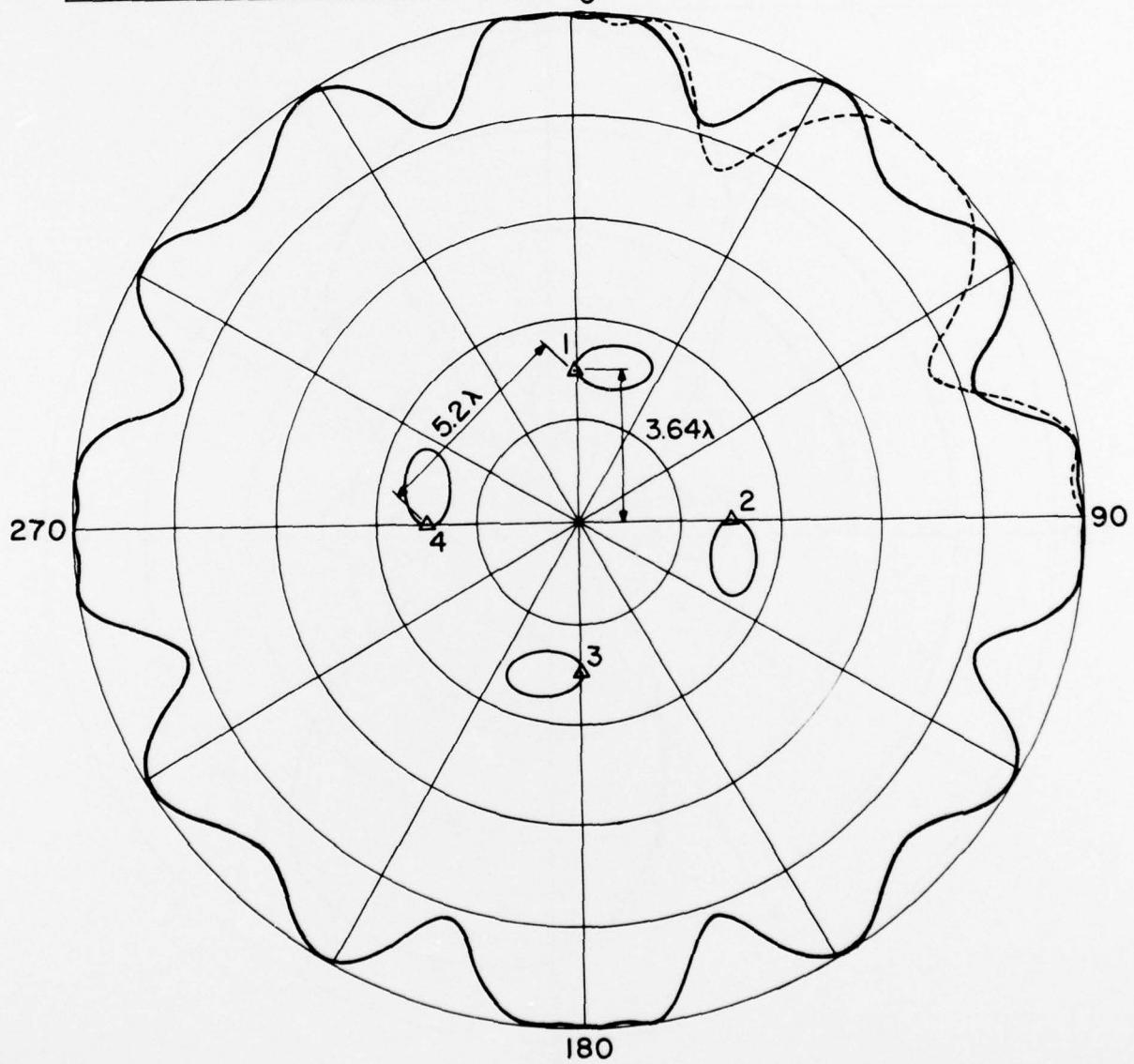


Fig. 10 - Skew Array -  $5\lambda$  - No Back Lobe

i	1	2	3	4
$I_i$	1.051	1.051	1.051	1.051
$\alpha_i$	0°	0°	0°	0°
$\phi_i$	0°	90°	180°	-90°
$\gamma_i$	90°	180°	-90°	0°
$d_i/\lambda$	3.64	3.64	3.64	3.64

----- THEORETICAL  
— OPTIMIZED

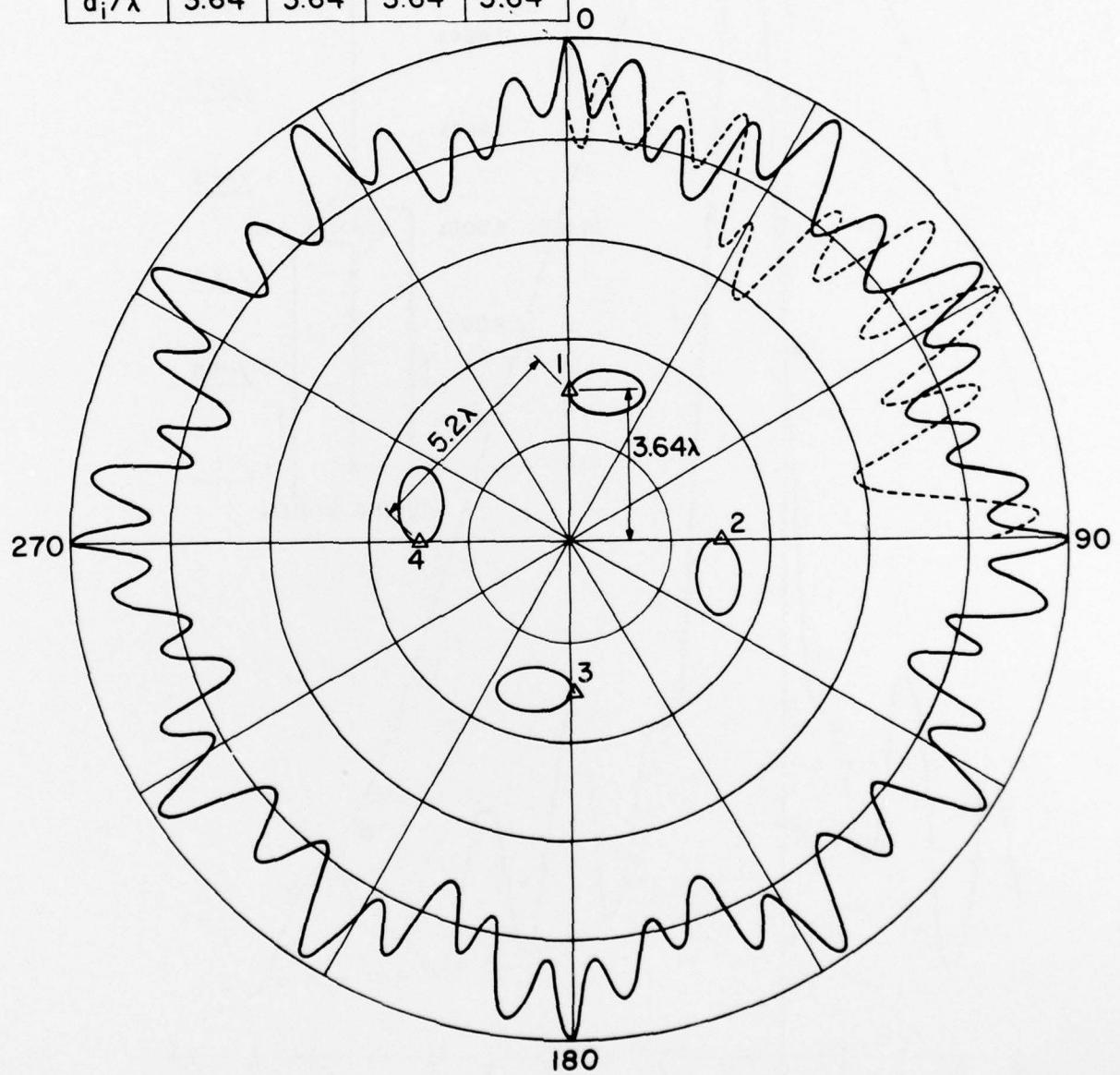


Fig. 11 - Skew Array -  $5\lambda$  - With Back Lobe

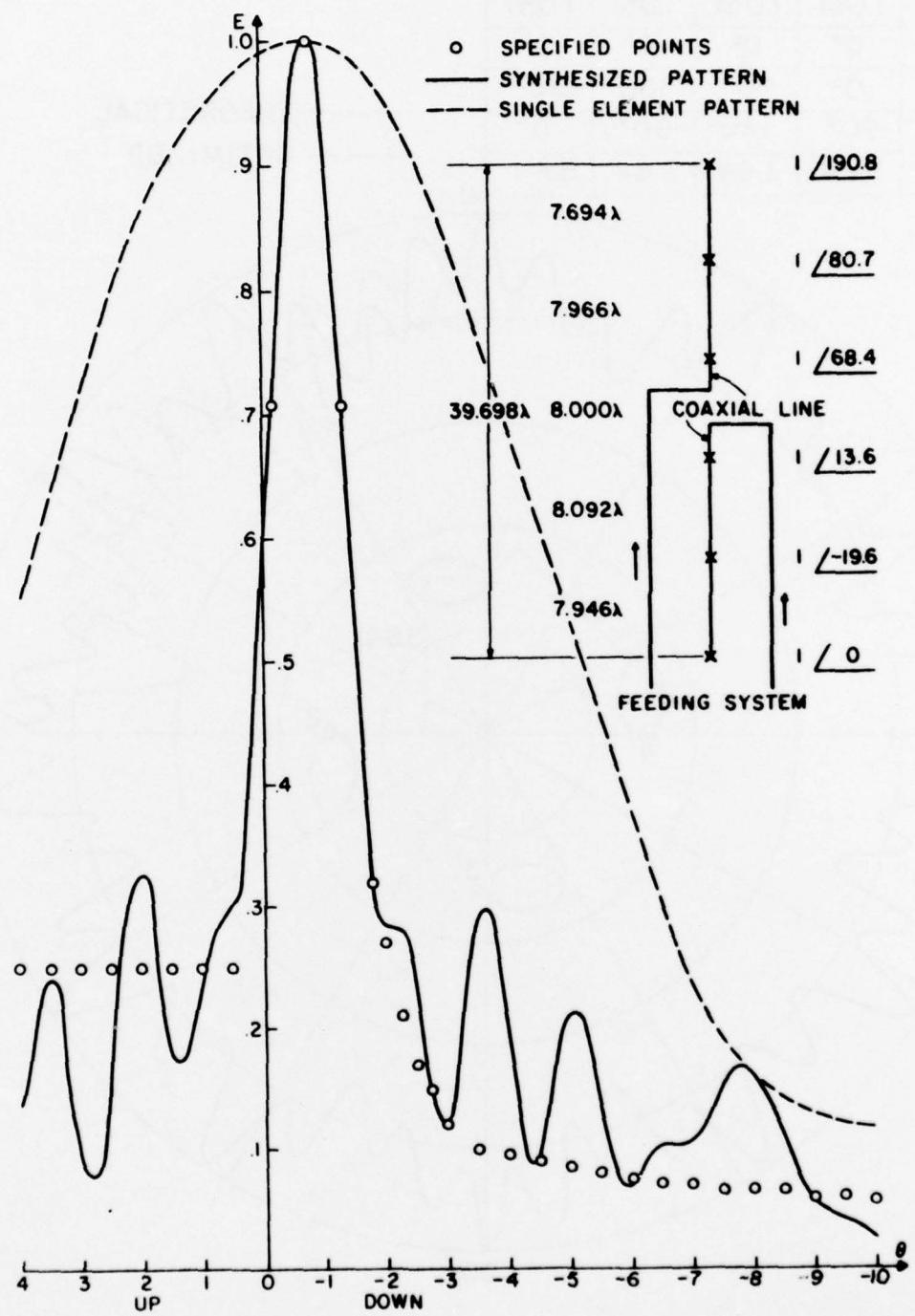


Fig. 12 - TV Antenna Vertical Pattern With Equal Amplitudes

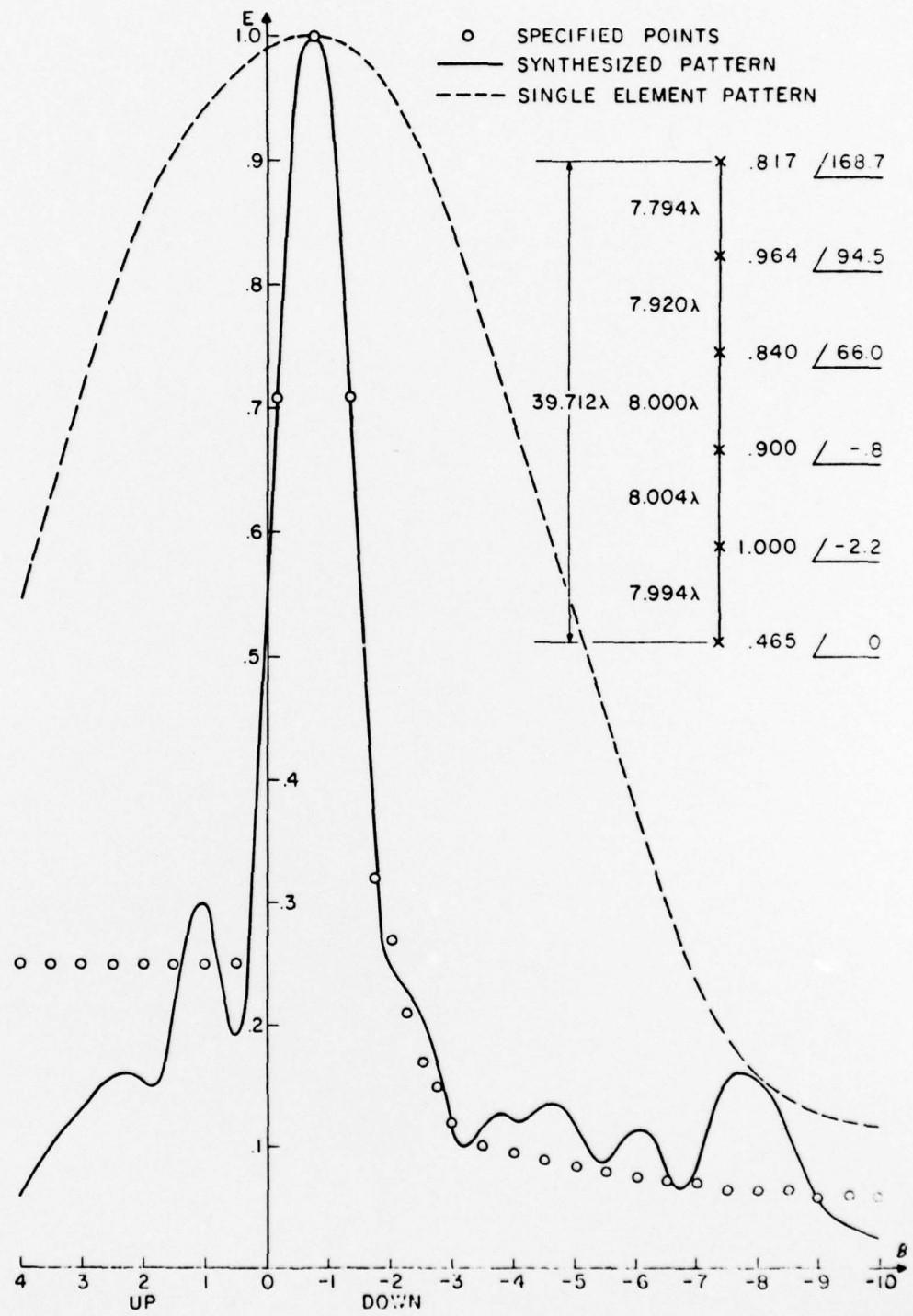


Fig. 13 - TV Antenna Vertical Pattern With Variable Amplitudes

## APPENDIX

## Computer Program Flow Chart and Typical Listing

A flow chart of the computer program used for the solution of some of the examples presented in this paper is shown in Fig. 14. An explanation follows making reference to the numbered boxes of the chart:

1 - Certain statements of the program have to be changed to specify different constraints. For example we may desire that the amplitudes be greater than .8. When any amplitude reaches a value smaller than that it is set equal to .8.

2 - The weighting function  $W(\phi)$  is different for each problem and therefore must be specified when a new problem is being worked out.

3 - The program always prints the specified pattern so that a check on its accuracy can be made before the synthesis is started.

4 - The computer will ask the values of the five variables specified here.

If:  $JGO = 1$  no output is provided.

$JGO = 2$  the array parameters are printed.

$JGO = 3$  a plot of both the specified and the synthesized patterns is provided.

$JGO = 4$  the synthesized pattern is printed in a table form.

NUM is the number of desired iterations.

$C1, C2, C3$  are the gain factors for the amplitudes, phases and spacings respectively.

5,6,7 - If C1 or C2 or C3 is equal to zero no iterations on the corresponding variable is performed. This is another way of introducing constraints in the problem. If C1 is zero, for example, the amplitudes will be equal to the initial guess throughout the synthesis.

8 - The switch JGO performs the functions explained in 4.

The program developed by the authors consists of a series of subroutines or functions that can readily be changed to accomodate a new problem. Numerical computation of derivatives has been used throughout. If one desires to use the actual equations to compute the gradient components, all that is required is the writing of a new subroutine.

In the end of this Appendix there is a listing of a program which has been specially tailored to the case of synthesis of linear arrays.

The following notation was used in the program:

M	error after each iteration
N	number of elements
NN	number of specified points
C	$2\pi/360$
CC	$2\pi$
X(1) to X(10)	amplitude of each element
X(11) to X(20)	phase of each element
X(21) to X(30)	position of each element referred to the origin
TET(I)	specified angles
SPEC(I)	specified field strength
DX(I)	derivatives of M with respect to each element in same order as X(I).

The program is composed of the following parts:

Lines 00010 - 00110 Computation of Field Strength

In this subroutine the field strength is evaluated. The routine is called for an angle TETX and returns the value of the real part of the field under the name FRE, the imaginary under FIM and the absolute value under RES.

Lines 00130 - 00270 Computation of the Weighted Least Squares

In this part the weight function W is defined and the value of M (called error E) is computed. The formulation given in the listing, is that for a 4 element Chebyschev array with 20 dB sidelobe level.

Lines 00290 - 00390 Computation of the Gradient

Numerical calculation of the partial derivative of M with respect to each parameter which is a member of the group denoted by NC.

Lines 00410 - 00530 Alternative Computation of the Gradient for Constant Group

This subroutine is called, instead of the above, when every parameter of the whole group, for example amplitudes, is restricted to have the same value, although this value has to be iterated.

Lines 00550 - 00610 Iteration

New parameter setting according to Equation (7). Here restrictions on the magnitude of the variables can be introduced as seen on line 00600.

Lines 00630 - 00690 Alternative Iteration for Constant Group

New Parameter setting for the special case referred to above.

Lines 00710 - 00910 Pattern Output Graph

Use has been made of a library routine called PLOT. Both patterns, the current and the specified one, are plotted.

Lines 01090 - 01330 Specification and Data for the Main Program

This part of the program is concerned with format statements and with the input data.

Lines 01350 - 01680 Main Program

Connects all the subroutines and functions as suggested in the flow chart.

```

00010 * COMPUTATION OF FIELDSTRENGTH
00020 SUBROUTINE FUNCT(TETX,FRE,FIM,RES)
00030 COMMON N,NN,C,CC,X(30)
00040 FRE=0.; FIM=C.
00050 STET=SIN(C*TETX)
00060 DO 30 I=1,N
00070 FI=C*X(I+10)+CC*X(I+20)*STET
00080 FRE=FRE+X(I)=COS(FI)
00090 30 FIM+FIM+X(I)*SIN(FI)
00100 RES=SQRT(FRE**2+FIM**2)
00110 RETURN;END
00120 *
00130 * COMPUTATION OF WEIGHTED LEAST SQUARES
00140 SUBROUTINE ERR(E)
00150 COMMON N,NN,C,CC,X(30),TET(64),SPEC(64)
00160 E=0
00170 R1=15.;R2=-15.
00180 DO 10 J=1,NN
00190 CALL FUNCT(TET(J),FRE,FIM,RES)
00200 DIFF=RES-SPEC(J)
00210 IF (TET(J) .GE. R1 .AND. DIFF .LE. 0.) W=0.
00220 IF (TET(J) .GE. R1 .AND. DIFF .GT. 0.) W=1.
00230 IF (TET(J) .LE. R2 .AND. DIFF .LE. 0.) W=0.
00240 IF (TET(J) .LE. R2 .AND. DIFF .GT. 0.) W=1.
00250 IF (TET(J) .LT. R1 .AND. TET(J) .GT R2) W=5.
00260 10 E=E+W*(DIFF**2)
00270 RETURN; END
00280 *
00290 * COMPUTATION OF THE GRADIENT
00300 SUBROUTINE DERIV (NC,E1)
00310 COMMON N,NN,CNCC,X(30),TET(64),SPEC(64),DX(30)
00320 CO=1.005
00330 DO 1 J=1,N
00340 K=J+NC
00350 X(K)=X(K)*CO
00360 CALL ERR(E2)
00370 DX(K)=(E2-E1)/(X(K)*(CO-1.))
00380 1 X(K)=X(K)/CO
00390 RETURN;END
00400 *
00410 * ALTERNATIVE COMPUTATION OF GRADIENT FOR CONSTANT GROUP
00420 SUBROUTINE DERIV2(NC,E1)
00430 COMMON N,NN,C,CC,X(30),TET(64),SPEC(64),DX(30)
00440 X(1)=1.005*X(1)
00450 DO 1 J=2,N
00460 1 X(J)=X(1)
00470 CALL ERR(E2)
00480 DX(1)(E2-E1)/(X(1)*.005)
00490 X(1)=X(1)/1.005
00500 DO 2 J=2,N
00510 X(J)=X(1)
00520 2 DX(J)=DX(1)
00530 RETURN;END

```

```

00540 *
00550 * ITERATION PROCEDURE
00560 SUBROUTINE ITER(NC,CK)
00570 COMMON NMNNMCMCCX(30),TET(64),SPEC(64),DX(30)
00580 DO 1 J=1,N
00590 K=J+NC
00598 X(K)+X(K)-CK*DX(K)
00600 1 IF (NC .EQ. 0 .AND.X(K) .LT. .09) X(K)=.09
00610 RETURN;END
00620 *
00630 * ALTERNATIVE ITERATION FOR CONSTANT GROUP
00640 SUBROUTINE ITER2(NC,CK)
00650 COMMON N,NN,C,CC,X(30),TET(64),SPEC(64),DX(30)
00660 X(1)=X(1)-CK*DX(1)
00670 DO 1 J=2,N
00680 1 X(J)=X(1)
00690 RETURN;END
00700 *
00710 * PATTERN OUTPUT - GRAPH
00720 SUBROUTINE GRAPH
00730 COMMON N,NN,C,CC,X(30),TET(64),SPEC(64)
00740 DIMENSION PR(2)
00750 TETX=95.
00760 NNN=37
00770 CALL PLOT(TETX,PR,1.2,-.2,2,1,NNN)
00780 DO 12 J=1,NNN
00790 TETX=TETX-.5.
00800 CALL FUNCT(TETX,FRE,FIM,RES)
00810 PR(2)=0.
00820 PR(2)=RES
00830 DO 13 I=1,NN
00840 IF (TETX-TET(I)) 13,14,13
00850 14 PR(2)=SPEC(I)
00860 CALL PLOT(TETX,PR,1.2,-.2,2,0,NNN)
00870 GO TO 12
00880 13 CONTINUE
00890 CALL PLOT(TETX,PR,1.2,-.2,2,0,NNN)
00900 12 CONTINUE
00910 RETURN;END
00920 *
00930 I PATTERN OUTPUT - DATA
00940 SUBROUTINE DATA
00950 COMMON N,NN,C,CC,X(30),TET(64),SPEC(64)
00960 56 FORMAT(I3,6G11.3)
00970 TETX=92.
00980 PRINT:"PATTERN DATA"
00990 DO 20 J=1,50
01000 TETX=TETX-2
01010 CALL FUNCT(TETX,FRE,FIM,RES)
01020 IF (FRE*FIM) 3,4,3
01030 4 PH=0.
01040 GO TO 20
01050 3 PH=ATAN2(FIM,RES)/C
01060 20 PRINT 56, J,TETX,PH,RES

```

```

01080 *
01090 * SPECIRICATION AND DATA FOR MAIN PROGRAM
01100 COMMON N,NN,C,CC,X(30),TET(64),SPEC(64),DX(30)
01110 50 FORMAT (8H ERROR= ,E11.3)
01120 51 FORMAT (8H ERROR= ,E11.3,10H DIFF%= ,G9.2)
01130 52 FORMAT (7H AMPL= ,6G11.4)
01140 53 FORMAT (7H PHAS= ,6G11.4)
01150 54 FORMAT (7H DAMP= ,6G11.4)
01160 55 FORMAT (7H DPHA= ,6E11.4)
01170 56 FORMAT (I3,6G11.3)
01180 57 FORMAT (-H ,28H ENTER: JOB,ITER#,C1,C2,C3 )
01190 58 FORMAT (7H DIST= ,6G11.4)
01200 59 FORMAT (7H DDIS= ,6G11.4)
01210 N=8;NN=35
01220 PRINT:"X"
01230 READ:(X(I),II=1,N),(X(I+10),I=1,N),(X(I+20),I=1,N)
01240 PRINT:"TET"
01250 READ:(TET(I),I=1,NN)
01260 PRINT:"SPEC"
01270 READ:(SPEC(I),I=1,NN)
01280 PRINT:"SPECIFICATIONS"
01290 DO 5 J=1,NN
01300 5 PRINT 56,J,TET(J),SPEC(J)
01310 PI=3.14159265
01320 CC=2.*PI
01330 C=CC/360.
01340 *
01350 * MAIN PROGRAM
01360 DO 2 J=1,30
01370 2 DX(J)=0.
01380 CALL ERR(E1)
01390 PRINT 50,E1
01400 10 PRINT 57
01410 READ:JGO,NUM,C1,C2,C3
01420 IF (JGO .EQ. 99) GO TO 99
01430 IF (NUM .EQ.0) GO TO 7
01440 6 DO 1 J=1,NUM
01450 IF (C1 .NE. 0.) CALL DERIV(0,E1)
01460 IF (C2 .NE. 0.) CALL DERIV(10,E1)
01470 IF (C3 .NE. 0.) CALL DERIV(20,E1)
01480 IF (C1 .NE. 0.) CALL ITER (0,C1)
01490 IF (C2 .NE. 0.) CALL ITER(10,C2)
01500 IF (C3 .NE. 0.) CALL ITER(20,C3)
01510 CALL ERR(E2)
01520 PRINT 51, E2,100*(E1-E2)/E2
01530 1 E1=E2
01540 7 GO TO (10,8,11,18),JGO
01550 8 PRINT:"STATUS"
01560 PRINT 52,(X(I),=1,N)
01570 PRINT 53,(X(I+10),I=1,N)
01580 PRINT 58,(X(I+20),I=1,N)
01590 PRINT 54,(DX(I),I=1,N)
01600 PRINT 55,(DX(I+10),I=1,N)
01610 PRINT 59,(DX(I+20),I=1,N)
01620 GO TO 10
01630 11 CALL GRAPH

```

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Ø164Ø GO TO 1Ø  
Ø165Ø 18 CALL DATA  
Ø166Ø GO TO 1Ø  
Ø167Ø 99 STOP  
Ø168Ø END

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## METRIC SYSTEM

### BASE UNITS:

<u>Quantity</u>	<u>Unit</u>	<u>SI Symbol</u>	<u>Formula</u>
length	metre	m	...
mass	kilogram	kg	...
time	second	s	...
electric current	ampere	A	...
thermodynamic temperature	kelvin	K	...
amount of substance	mole	mol	...
luminous intensity	candela	cd	...

### SUPPLEMENTARY UNITS:

plane angle	radian	rad	...
solid angle	steradian	sr	...

### DERIVED UNITS:

Acceleration	metre per second squared	...	m/s
activity (of a radioactive source)	disintegration per second	...	(disintegration)/s
angular acceleration	radian per second squared	...	rad/s
angular velocity	radian per second	...	rad/s
area	square metre	...	m <sup>2</sup>
density	kilogram per cubic metre	...	kg/m <sup>3</sup>
electric capacitance	farad	F	A·s/V
electrical conductance	siemens	S	A/V
electric field strength	volt per metre	...	V/m
electric inductance	henry	H	V·s/A
electric potential difference	volt	V	W/A
electric resistance	ohm	...	V/A
electromotive force	volt	V	W/A
energy	joule	J	N·m
entropy	joule per kelvin	...	J/K
force	newton	N	kg·m/s
frequency	hertz	Hz	(cycle)/s
illuminance	lux	lx	lm/m <sup>2</sup>
luminance	candela per square metre	...	cd/m <sup>2</sup>
luminous flux	lumen	lm	cd·sr
magnetic field strength	ampere per metre	...	A/m
magnetic flux	weber	Wb	V·s
magnetic flux density	tesla	T	Wb/m
magnetomotive force	ampere	A	...
power	watt	W	J/s
pressure	pascal	Pa	N/m
quantity of electricity	coulomb	C	A·s
quantity of heat	joule	J	N·m
radiant intensity	watt per steradian	...	W/sr
specific heat	joule per kilogram-kelvin	...	J/kg·K
stress	pascal	Pa	N/m
thermal conductivity	watt per metre-kelvin	...	W/m·K
velocity	metre per second	...	m/s
viscosity, dynamic	pascal-second	...	Pa·s
viscosity, kinematic	square metre per second	...	m <sup>2</sup> /s
voltage	volt	V	W/A
volume	cubic metre	...	m <sup>3</sup>
wavenumber	reciprocal metre	...	(wave)/m
work	joule	J	N·m

### SI PREFIXES:

Multiplication Factors	Prefix	SI Symbol
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	T
$1\ 000\ 000\ 000 = 10^9$	giga	G
$1\ 000\ 000 = 10^6$	mega	M
$1\ 000 = 10^3$	kilo	k
$100 = 10^2$	hecto*	h
$10 = 10^1$	deka*	da
$0.1 = 10^{-1}$	deci*	d
$0.01 = 10^{-2}$	centi*	c
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001 = 10^{-9}$	nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	p
$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$	femto	f
$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$	atto	a